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# Subsidies Versus Public Provision of Private Goods as Instruments for Redistribution

Robin Boadway  
Queen's University

Maurice Marchand  
Universite Catholique de Louvain

Motohiro Sata  
Queen's University

Department of Economics  
Queen's University  
94 University Avenue  
Kingston, Ontario, Canada  
K7L 3N6

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**Robin Boadway, Queen's University**  
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## **Abstract**

The literature on the use of differential commodity taxes/subsidies and that on quantity controls to supplement income taxation have developed separately from each other. The purpose of this paper is to combine these two strands in the standard framework of optimal non-linear income taxation. We start from a simple model in which there are two types of households, the government has access to both subsidy policy and public provision of a good substitutable with leisure, and households can supplement the publicly provided good from the market. We present conditions when optimal policy should involve a mix of these two instruments alongside income taxation or only one of them. We also consider alternative settings, including the extension to many types of households and the inability of households to supplement in-kind transfers.

KEY WORDS: in-kind transfers, subsidies, optimal income tax

JEL CLASSIFICATION: H2, H4

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## 1. INTRODUCTION

Recent literature on redistribution policy has focused on efficiency aspects of redistribution processes. It has sought to determine ways in which the economy's second-best utility possibilities frontier can be extended as far as possible. It is now widely recognized that a major constraint facing policy-makers in designing an efficient redistribution policy is an informational one: governments are not as well-informed about the relevant utility-determining characteristics of taxpayers as the taxpayers themselves are. Governments that attempt to redistribute from the better-off to the less well-off are faced with incentive-compatibility constraints that severely limit the amount of redistribution that can be accomplished.<sup>1</sup> When informationally based incentive constraints are viewed as limits to redistribution, it is natural to consider alternative policy instruments in addition to income taxation which might serve to relax those constraints.

Two approaches have been taken in searching for policy instruments to complement income taxation — one based on pricing instruments and the other on quantity controls. The pricing instruments involve using commodity taxes alongside redistributive income taxes. The use of a mix of direct and indirect taxes as part of an optimal tax policy has been a traditional issue addressed in the optimal tax literature.<sup>2</sup> But it is only recently that the structure of commodity taxes has been analyzed from the perspective of improving the efficiency properties of tax policy by relaxing the incentive constraints. Edwards *et al* (1994) and Nava *et al* (1996) have shown how the commodity tax structure ought to be designed to improve the efficiency of redistributive tax policy. For example, by imposing relatively low (high) commodity tax rates on goods that are substitutable (complementary) with leisure, the incentive constraint applying on better-off households can be relaxed and Pareto improvements can be achieved. A very general treatment of the use of a mix of direct and indirect taxes in designing redistributive policy mechanisms in the face of informational constraints is provided by Guesnerie (1995).

Parallel to this work on the direct-indirect tax mix has been some recent work on using quantity-based policies alongside income taxes. The possibility that quantity controls

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<sup>1</sup> See, for example, Roberts (1980), Stiglitz (1987), Tuomala (1990) and Guesnerie (1995) for representative discussions of this. Of course, this approach originates in Mirrlees (1971).

<sup>2</sup> The seminal piece is Atkinson and Stiglitz (1976). The tax mix problem is treated fully in Atkinson and Stiglitz (1980).

might be efficient policy instruments owes its origin to the seminal paper by Guesnerie and Roberts (1984). They argued that in a second-best setting, that is, in an economy with price distortions, quantity controls will generally be welfare-improving, provided they cannot be circumvented by reselling. They could take the form of forced consumption (in-kind transfers) or restricted consumption (rationing). A series of recent papers have extended the Guesnerie-Roberts analysis to the case where the source of the distorted economy could be traced to informational constraints and where the government was otherwise fully exploiting the use of an optimal non-linear income tax as a policy instrument. For example, Boadway and Marchand (1995) and Blomquist and Christiansen (1996) have shown that in-kind transfers can improve the efficiency of redistributive policy even when the government is employing an optimal non-linear income tax. In-kind transfers of a good that cannot be retraded will be efficiency-enhancing if the good is a substitute for leisure.<sup>3</sup>

The literature on the use of commodity taxes to relax the incentive constraints and that on quantity controls to relax the incentive constraints have evolved separately from each other. One of the interesting feature to come out of these two streams of work is that the same conditions that make it efficient to tax a particular good preferentially also make it efficient to provide in-kind transfers of that good when an optimal non-linear income tax is in place. A natural issue to address is what happens if policy-makers are allowed to use either commodity taxes or in-kind transfers, or some combination of the two. Are price-based controls preferable to quantity-based controls, or vice versa? Or are they complementary instruments? The purpose of this paper is to consider that issue. We derive conditions under which it would be optimal to employ either type of instrument alone or together as part of a policy mix.

The model we use to address this issue is a straightforward extension of the standard self-selection approach to redistributive tax policy proposed by Stiglitz (1982) and others.<sup>4</sup> We first develop the simple case in which there are two types of households, low- and high-ability, who consume two goods and leisure. We suppose that one of the goods cannot be retraded and is substitutable with leisure, precisely the conditions under which either preferential commodity taxation or in-kind transfers are welfare-improving. It is also initially assumed that households can supplement the in-kind transfers with private purchases. Having addressed the above issues for this case, we then consider alternative settings. These include the extension to many types of households and the inability of households to supplement in-kind transfers with private purchases.

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<sup>3</sup> Other examples include of quantity-based policies that can improve the efficiency of the redistributive process include minimum wages (Marceau and Boadway, 1994) and workfare (Besley and Coate, 1995).

<sup>4</sup> Various other authors recognized the fact that optimal tax policy could be seen as a problem of mechanism design in an asymmetric information environment, including Guesnerie and Seade (1982), Stern (1982) and Nichols and Zeckhauser (1982).

## 2. THE MODEL

The analysis can be developed by employing a simple extension of the economy used by Stiglitz (1982) to characterize the optimal non-linear income tax problem. The economy consists of two types of households who differ only in their abilities to earn incomes, which are fully reflected in their wage rates  $w_1$  and  $w_2$ , where  $w_1 < w_2$ . There are three commodities in the economy — two goods,  $x$  and  $z$ , and labour,  $\ell$ . The numeraire is  $x$  and the producer price of  $z$  is  $p$ . Labour is the only factor of production, markets are competitive, and producer prices  $(p, w_1, w_2)$  are fixed. Households supply labour and incur an income tax on their labour income. They spend their after-tax income on the two goods. The government may also provide some of the good  $z$  to all households free of charge. Initially, we assume that the government supplies the same amount  $g$  to all households. Households may supplement public provision by their own purchases, denoted  $b$ , but good  $z$  cannot be retraded, so  $z = b + g$  with  $b \geq 0$ . Subsequently, we allow for the fact that public provision to an individual may be mutually exclusive with private purchases. In this case, households must choose whether to opt for public provision or opt out and provide for themselves. Both types of assumptions have been used in the literature.

In addition to imposing a non-linear income tax and providing  $g$  to all households, the government can also employ a system of differential commodity taxes, that is a set of taxes on  $x$  and  $z$ . Given that an income tax exists alongside the commodity taxes, the only relevant feature of the commodity tax structure is the *relative* tax rates on goods  $x$  and  $z$ . The absolute level of commodity taxes is indeterminate since a proportional increase in goods tax rates is equivalent to a comparable increase in income tax liabilities on the two types of households. Therefore, there is no loss in generality in restricting the commodity tax structure to be one in which the tax rate on good  $x$  is zero. We characterize the commodity tax structure by the per unit subsidy  $s$  on good  $z$  so the consumer price is  $p - s \equiv \bar{p}$ .

This set of government policy instruments — a non-linear income tax combined with commodity taxes and public provision of  $z$  — reflect the informational constraints facing the government. We adopt the same informational assumptions as Guesnerie (1995). The government is assumed to be able to observe incomes, but not wage rates or labour supplies. Transactions in  $x$  and  $z$ , but not the identity of purchasers, are observable. For example, firms might be responsible for collecting and remitting taxes. We also assume that public provision of the private good  $z$ , which is not tradable, cannot be contingent on income; for example, public provision occurs before incomes are reported.<sup>5</sup> The government does know the proportions of the population that are high- and low-ability, and knows the form of the household utility functions.

All households have identical concave utility functions,  $U(x, z, \ell)$ . Let  $Y_i = w_i \ell_i$  denote labour income earned by a household of type  $i$ . Using this, the household's utility function

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<sup>5</sup> This corresponds with what others have assumed in the literature on the public provision of private goods, including Guesnerie and Roberts (1984), Boadway and Marchand (1995), Munro (1992), Blomquist and Christiansen (1995) and Guesnerie (1995).

may be written:

$$U^i(x, z, Y) \equiv U\left(x, z, \frac{Y}{w_i}\right)$$

with  $z = b + g$ . The utility function is increasing in  $x$  and  $z$ , decreasing in  $Y$ . We assume that  $x$ ,  $z$  and leisure are all normal goods, and below we assume that  $z$  and leisure are substitutes in the sense that, at given consumer prices and disposable income, the demand for  $z$  falls as more leisure becomes available.<sup>6</sup> The household maximizes utility by choosing  $x$ ,  $b$  and  $Y$  subject to a budget constraint of the form  $x + \bar{p}b = Y - T(Y)$ , where  $z = b + g$  and  $g$  is given, and where  $T = T(Y)$  is the non-linear tax function. For notational ease, we often write  $T_i \equiv T(Y_i)$ .

It is convenient to disaggregate a household's problem into two stages. In one stage, the household chooses its labour supply  $\ell$  and therefore its before-tax income  $Y = w\ell$ . In the other, it spends its after-tax income on the two goods  $x$  and  $b$ . We begin with the latter stage. Let  $B_i = Y_i - T_i$  be the disposable income of a household of type  $i$ . Given  $Y_i$  and thus  $B_i$ , household  $i$ 's problem can be written as:

$$\max_{\{b \geq 0\}} U^i(B_i - \bar{p}b, b + g, Y_i)$$

The Kuhn-Tucker first-order conditions may be written:

$$-\bar{p}U_x^i + U_z^i \leq 0; \quad (-\bar{p}U_x^i + U_z^i)b = 0 \quad (1)$$

The solution to this problem yields the demand function  $b_i(\bar{p}, g, Y_i, B_i)$ . There are two possible outcomes, one in which the non-negativity constraint on  $b$  is binding, and one in which it is not.

In the case in which  $b_i > 0$ , the following properties apply to the demand function  $b_i(\bar{p}, g, Y_i, B_i)$ :

$$(i) : \frac{\partial b_i}{\partial B_i} > 0; \quad (ii) : \frac{\partial b_i}{\partial \bar{p}} = \frac{\partial \tilde{b}_i}{\partial \bar{p}} - b_i \frac{\partial b_i}{\partial B_i} < 0; \quad (iii) : \frac{\partial b_i}{\partial g} < 0; \quad (iv) : \frac{\partial b_i}{\partial Y_i} > 0.$$

Property (i) reflects the assumption that good  $z$  is a normal good. Property (ii) is simply the Slutsky decomposition for this case, where  $\tilde{b}_i(\bar{p}, g; Y_i, U^i)$  is the compensated demand for good  $z$  by household  $i$ . The interpretation of property (iii) is as follows. If  $b_i > 0$ , public provision  $g$  is equivalent to a lump-sum transfer of  $\bar{p}g$ : it has only an income effect on the household's behaviour. An increase in  $g$  increases the household's overall demand for  $z$  by an income effect, but reduces the demand for  $b$  since  $g$  and  $b$  are perfect substitutes, so  $\partial b_i / \partial g = \bar{p} \partial b_i / \partial B_i - 1$ . Since  $x$  is a normal good, the marginal propensity to spend disposable income on  $b_i$ , the first term, is less than unity, so property (iii) follows.

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<sup>6</sup> Equivalently,  $\partial MRS_{zx}^i / \partial Y_i > 0$ , where  $MRS_{zx}^i$  is the marginal rate of substitution of  $z$  for  $x$  in the utility function  $U^i(x, z, Y)$ . With  $x$  and  $z$  normal,  $z$  and leisure are substitutable in this sense if they are Hicksian substitutes.

Finally, property (iv) reflects the assumption that good  $z$  is a substitute for leisure. This assumption assures both that the optimal commodity tax structure (with  $g = 0$ ) will be one in which  $s > 0$  (Edwards *et al* (1994), Nava *et al* (1996)) and that public provision (with  $s = 0$ ) will be welfare-improving (Boadway and Marchand, 1995). This provides the context for investigating the appropriate mix of subsidy and public provision.

The relationship between the values of  $b_1$  and  $b_2$  chosen by the household will be relevant for the analysis to follow. In principle, the demand for commodity  $z$  could be greater for either the high-ability or the low-ability households (assuming that the non-negativity constraint is not binding) depending on pairs,  $(Y_i, B_i), i = 1, 2$ . But given that  $z$  is normal and substitutable for leisure, it is straightforward to show that  $b_2(\bar{p}, g, Y_2, B_2) > b_1(\bar{p}, g, Y_1, B_1)$  if  $\ell_2 > \ell_1$  and  $B_2 > B_1$ .<sup>7</sup> As we will see,  $B_2 > B_1$  with  $Y_2 > Y_1$  in the optimal tax equilibrium. And we will argue that under reasonable conditions,  $\ell_2 > \ell_1$ . For that reason, we will regard  $b_2 > b_1$  as being the normal case in subsequent sections.

The maximum value function for problem (1) is denoted

$$V^i(\bar{p}, g, Y_i, B_i) \equiv U^i(B_i - \bar{p}b(\bar{p}, g, Y_i, B_i), b(\bar{p}, g, Y_i, B_i) + g, Y_i) \quad (2)$$

By the envelope theorem, it has the following properties when  $b_i > 0$ :

$$\frac{\partial V^i}{\partial \bar{p}} = -U_x^i b_i; \quad \frac{\partial V^i}{\partial B_i} = U_x^i; \quad \frac{\partial V^i}{\partial g} = \bar{p}U_x^i = U_z^i; \quad \frac{\partial V^i}{\partial Y_i} = U_Y^i \quad (3)$$

If the level of  $g$  is such that the household's purchases of  $z$  are fully crowded out ( $b_i = 0$ ), the household has no effective discretion. All disposable income  $B_i$  is spent on good  $x$ . The 'indirect' utility function becomes simply  $V^i(\bar{p}, g, Y_i, B_i) = U^i(B_i, g, Y_i)$ . The envelope properties for  $B_i$  and  $Y_i$  in (3) still apply. As well,  $\partial V^i / \partial g = U_z^i$  and  $\partial V^i / \partial \bar{p} = 0$ .

The indirect utility function  $V^i(\bar{p}, g, Y_i, B_i)$  indicates the utility that the household would obtain from given levels of before- and after-tax income if it spent its income optimally. The indirect utility function could be depicted geometrically by an indifference map in  $(B_i, Y_i)$ -space. The slope of the indifference curve will be the marginal rate of substitution between  $B_i$  and  $Y_i$ ,  $MRS_{YB}^i = -\frac{\partial V^i / \partial Y_i}{\partial V^i / \partial B_i} > 0$ . It is straightforward to show using the properties of the utility function that  $MRS_{YB}^i$  is increasing as one moves up an indifference curve, and that  $b_i$  decreases when moving down an indifference curve (until the household is crowded out). What is of more relevance is the single-crossing or Spence-Mirrlees condition, which in this context is that  $MRS_{YB}^1 > MRS_{YB}^2$ , so indifference curves of the low-ability households are steeper than those of the high-ability households. In problems of this sort, there is no guarantee that it will hold. Following Stiglitz (1987) and virtually all of the subsequent literature, we simply assume that the single-crossing property holds.

The choice of a  $(Y_i, B_i)$  combination to maximize  $V^i(\cdot)$  constitutes the first stage of the household's problem. The selection of a non-linear tax schedule by the government can

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<sup>7</sup> To see this, note that  $b_i(\bar{p}, g, Y_i, B_i)$  can be written as  $b(\bar{p}, g, Y_i/w_i, B_i) = b(\bar{p}, g, \ell_i, B_i)$  for both types of households.

be depicted as the offering of a menu of  $(B_i, Y_i)$  alternatives to the households. In offering this menu, the government must rely on households to self-select, that is, for household  $i$  to choose the  $(B_i, Y_i)$  combination intended for it. To induce this self-selection, the menus offered must be such that neither ability-type household prefers the  $(B_i, Y_i)$  combination meant for the other. This is the basis for the incentive or self-selection constraints imposed on the government's choice of income tax schedule. To specify the incentive constraints, it is necessary to analyze the behaviour of a 'mimicking' household, one that chooses the  $(B_i, Y_i)$  combination intended for the other ability-type. In this paper, we assume that the self-selection constraint is binding on the high-ability person. This implies that we are restricting our analysis to government objective functions that entail redistribution from high-to low-ability households such that the self-selection constraint binds.

A high-ability household that pretends to be low-ability will choose to earn an income of  $Y_1$  and receive an after-tax income of  $B_1$ . Denoting variables for the mimicker with a hat (^), the labour supply of the mimicker must therefore be:

$$\hat{\ell}_2 = \frac{Y_1}{w_2} < \frac{Y_1}{w_1} = \ell_1 \quad (4)$$

The mimicker's problem is to choose  $b$  to maximize  $\hat{U}^2(B_1 - \bar{p}b, b + g, Y_1)$ , subject to  $b \geq 0$ , where  $Y_1$  and  $B_1$  are given and  $Y_1 \equiv w_2 \hat{\ell}_2$ . The first order condition to this problem is analogous to (1). The solution gives the demand function  $\hat{b}_2(\bar{p}, g, Y_1, B_1)$ . The maximum value function for the mimicker's problem is the indirect utility function  $\hat{V}^2(\bar{p}, g, Y_1, B_1)$ , with properties similar to (3).

Given our assumption that  $z$  is a substitute for leisure, it must be the case that for given values of  $x$  and  $z (= b + g)$ ,  $\widehat{MRS}_{zx}^2 < MRS_{zx}^1$ . Since the mimicker faces the same budget constraint as low-ability households, this in turn leads to:

$$\hat{b}_2(\bar{p}, g, Y_1, B_1) \leq b_1(\bar{p}, g, Y_1, B_1) \quad (5)$$

with  $\hat{b}_2 < b_1$  if the low-ability person is not crowded out by public provision ( $b_1 > 0$ ). Intuitively, since mimicking households consume more leisure than low-ability households, they will consume less of the substitute good  $z$ . Also, since  $Y_2 > Y_1$  and  $B_2 > B_1$ , and since  $b_i$  is increasing in both  $Y_i$  and  $B_i$  as long as  $b_i$  is not crowded out,

$$\hat{b}_2(\bar{p}, g, Y_1, B_1) \leq b_2(\bar{p}, g, Y_2, B_2) \quad (6)$$

with strict inequality holding if the high-ability person is not crowded out ( $b_2 > 0$ ).

The above results imply that as public provision  $g$  increases, the mimicker is crowded out before either the low-ability or high-ability households. It is this that justifies public provision of  $g$ : setting  $g$  at a level that crowds out the mimicker makes it more difficult to mimic and therefore relaxes the self-selection constraint. By the same token, in the absence of public provision, the fact that low-ability households demand more  $z$  than the mimicker also makes subsidization welfare improving. Both instruments can therefore enhance the



efficiency of the redistributive process. To see that, we turn to government policy. Recall that the government implements three sorts of policy instruments — a non-linear income tax, a subsidy on purchases of good  $z$ , and public provision of good  $z$  in equal amounts to all households. Our interest is in conditions under which the latter two instruments are used. Since we always assume that an optimal non-linear income tax is in place, we begin with a characterization of the form of this tax, given  $s$  and  $g$ .

### 3. OPTIMAL INCOME TAXATION

The government is assumed to use as an objective function the weighted sum of per capita utilities of the two ability types. Various weights will yield different points along the economy's utility possibilities frontier, given the constraints on the problem. As mentioned we restrict ourselves to points such that the self-selection constraint on the high-ability households is binding, so only those points on the utility possibilities frontier are considered. The government finances its subsidy on good  $z$  and its public provision of  $z$  using a non-linear income tax.

Given the size of the subsidy  $s$  and the level of public provision  $g$ , the government's optimal income tax problem may be written:

$$\max_{\{Y_i, B_i\}} V^1(\bar{p}, g, Y_1, B_1) + \lambda V^2(\bar{p}, g, Y_2, B_2) \quad (7)$$

subject to

$$V^2(\bar{p}, g, Y_2, B_2) \geq \hat{V}^2(\bar{p}, g, Y_1, B_1) \quad (\mu)$$

$$N_1(Y_1 - B_1 - sb_1 - pg) + N_2(Y_2 - B_2 - sb_2 - pg) \geq 0 \quad (\gamma)$$

where  $N_i$  is the number of ability-type  $i$  households in the economy and varying  $\lambda$  allows us to achieve various points on the Pareto efficient frontier. The first constraint is the incentive, or self-selection, constraint that rules out a high-ability household pretending to be low-ability. The second constraint is the government's budget constraint. The labels  $\mu$  and  $\gamma$  refer to the Lagrange multipliers that will be used for these two constraints. Both constraints are taken to be binding in what follows. This is a standard optimal non-linear tax problem. The first-order conditions are:

$$U_x^1 - \mu \hat{U}_x^2 - \gamma N_1 \left( 1 + s \frac{\partial b_1}{\partial B_1} \right) = 0 \quad (8)$$

$$(\lambda + \mu) U_x^2 - \gamma N_2 \left( 1 + s \frac{\partial b_2}{\partial B_2} \right) = 0 \quad (9)$$

$$U_Y^1 - \mu \hat{U}_Y^2 + \gamma N_1 \left( 1 - s \frac{\partial b_1}{\partial Y_1} \right) = 0 \quad (10)$$

$$(\lambda + \mu) U_Y^2 + \gamma N_2 \left( 1 - s \frac{\partial b_2}{\partial Y_2} \right) = 0 \quad (11)$$

For the high-ability person, combining (9) and (11) yields:

$$-\frac{U_Y^2}{U_x^2} = MRS_{YB}^2 = \frac{1 - s\partial b_2/\partial Y_2}{1 + s\partial b_2/\partial B_2}$$

This will be  $\leq 1$  for  $s \geq 0$ , in which case  $T'(Y_2) \geq 0$ . If  $s = 0$ , we obtain the standard result that the marginal tax rate on the high ability person is zero. With a positive subsidy in place, the marginal tax rates on the high-ability person is positive as long as  $b_2 \geq 0$  is not binding.

Combining (8) and (10), we can show that:<sup>8</sup>

$$-\frac{U_Y^1}{U_x^1} = MRS_{YB}^1 < 1$$

Defining the implicit marginal tax rate on the low-ability household (as in Stiglitz (1987)) by  $1 - T'(Y_1) = MRS_{YB}^1$ , we obtain the standard result that the marginal tax rate on the low-ability person is strictly positive,  $T'(Y_1) > 0$ .

With the optimal non-linear income tax and given values for  $s$  and  $g$ , households will choose their consumption bundles and labour supplies to maximize their utilities. It will be useful for us to know the relative magnitude of  $b_1$  and  $b_2$ . As discussed above,  $b_2 > b_1$  if  $B_2 > B_1$  and  $\ell_2 > \ell_1$ . When the optimal income tax is in place,  $B_2 > B_1$ . That is, high-ability households will have larger after-tax incomes than low-ability households. This well-known property of the optimal income tax follows from the self-selection constraint and the single-crossing property of indifference curves in  $(B, Y)$ -space. Moreover,  $\ell_2 > \ell_1$  if labour supply curves are upward-sloping ( $\ell_i$  is increasing in the after-tax wage rate) and if the subsidy is not too large.<sup>9</sup> Therefore, as mentioned earlier, we shall regard the case where  $b_2 > b_1$  when the households are not crowded out as the normal one in what follows.

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<sup>8</sup> To see this, add (8) and (10) to give:

$$U_x^1(1 + \frac{U_y^1}{U_x^1}) = \mu \hat{U}_x^2(1 + \frac{\hat{U}_y^2}{\hat{U}_x^2}) + \gamma s \{ \frac{\partial b_1}{\partial B_1} + \frac{\partial b_1}{\partial Y_1} \}$$

The first term on the RHS is positive since  $\widehat{MRS}_{YB}^2 < MRS_{YB}^2$  by concavity of the indifference curve and  $MRS_{YB}^2 \leq 1$  as just discussed. The second term is positive by the properties of the demand function,  $b_i(\cdot)$ .

<sup>9</sup> To see this, note that from the expression for  $MRS_{YB}^i$ , if  $s$  is not too large, the after-tax marginal wage rate for the high-ability person,  $(1 - T'(Y_2))w_2$ , will be greater than that of the low-ability person,  $(1 - T'(Y_1))w_1$ . If we linearize the budget constraint for the two types of households at the optimal income tax equilibrium, the high-ability household will have both a higher after-tax marginal wage and a lower virtual income. The latter will induce the high-ability household to supply more labour by the normality of leisure. And if the labour supply curve is upward sloping, the higher marginal wage rate will also induce more labour supply. Thus,  $\ell_2 > \ell_1$ .

The maximum value function for the government's optimal income tax problem (7), denoted  $W(s, g)$ , gives the value of social welfare that can be attained given the size of the subsidy  $s$  and the level of public provision  $g$ . By the envelope theorem, we obtain the following properties of  $W(s, g)$  after some manipulation and using the first-order conditions (8)–(11):

$$\frac{\partial W(s, g)}{\partial s} = \mu \hat{U}_x^2(b_1 - \hat{b}_2) + \gamma s \left( N_1 \frac{\partial \tilde{b}_1}{\partial \bar{p}} + N_2 \frac{\partial \tilde{b}_2}{\partial \bar{p}} \right) \quad (12)$$

and

$$\frac{\partial W(s, g)}{\partial g} = (U_z^1 - \bar{p}U_x^1) + (\lambda + \mu)(U_z^2 - \bar{p}U_x^2) - \mu(\hat{U}_z^2 - \bar{p}\hat{U}_x^2) - \gamma s(N_1 A_1 + N_2 A_2) \quad (13)$$

where

$$A_i = 1 - \bar{p} \frac{\partial b_i}{\partial B_i} + \frac{\partial b_i}{\partial g}.$$

From the properties of the demand function for  $b_i$  discussed above,  $A_i = 0$  if  $b_i > 0$  and  $A_i = 1$  if  $b_i = 0$ . We have also used the Slutsky equation for  $b_i$  in deriving (12).

Equations (12) and (13) provide the basis for analyzing the use of subsidies and public provision as government policies. We begin by considering each policy in turn, given the values of the other. Following that the optimal mix of the two policies is considered.

#### 4. THE WELFARE EFFECTS OF SUBSIDIES

We begin by considering the welfare effects of subsidies, given the level of public provision  $g$ . From (12), we immediately deduce the following proposition:

**Proposition 1:** *If  $z$  is a substitute for leisure, then:*

- i. at  $s \leq 0$ ,  $\partial W / \partial s > 0$  if  $b_1 > 0$ ;*
- ii. at  $s = 0$ ,  $\partial W / \partial s = 0$  if  $b_1 = 0$*

This proposition follows from the fact that  $b_1 \geq \hat{b}_2 \geq 0$ , with  $b_1 > \hat{b}_2$  if  $b_1 > 0$  by (5). Part *i.* implies that if low-ability persons are not crowded out, it will be welfare-improving to employ a subsidy on the purchase of  $z$ .<sup>10</sup> The reason is that a subsidy will be of more benefit to low ability households than to mimickers so will make mimicking more difficult. Introducing a subsidy financed by an income tax change that leaves both low- and high-ability households equally well off will reduce the utility of the mimicker. This will weaken the self-selection constraint and allow welfare to be improved. Part *ii.* states that if  $g$  is set at a level above that at which low-ability household are crowded out ( $b_1 = 0$ ), a subsidy will not be used in the optimum. If both the low-ability household and the mimicker are

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<sup>10</sup> Note also that if  $s < 0$  so that good  $z$  is taxed rather than subsidized,  $\partial W / \partial s > 0$  if  $b_1 > 0$ . That is, regardless of the value of  $g$ , as long as the low ability person is not crowded out, there should always be a subsidy rather than a tax on  $z$ . That implies that it will never be optimal to levy a tax on  $z$

crowded out, a subsidy cannot relax the self-selection constraint. All it will do is to distort the high-ability household's purchases of  $z$  if  $b_2 > 0$ .

If  $b_1 > 0$ , as  $s$  rises equation (12) becomes ambiguous in sign since the first term is positive while the second term is negative. If  $W(s, g)$  is strictly concave in  $s$  (but there is no guarantee that it will be),  $s$  will reach its global optimum at the level at which  $\partial W/\partial s = 0$ . Setting (12) equal to zero, we obtain the optimal subsidy  $s^*$ , given  $g$ , as:

$$s^*(g) = -\frac{\mu \hat{U}_x^2 (b_1 - \hat{b}_2)}{\gamma \left( N_1 \frac{\partial \hat{b}_1}{\partial \bar{p}} + N_2 \frac{\partial \hat{b}_2}{\partial \bar{p}} \right)} \quad (14)$$

This equation is similar to that derived by Edwards *et al* (1994) and Nava *et al* (1996), both for the case of  $g = 0$ . They provide an intuitive interpretation of the rules for differential commodity taxation that applies equally well here. Increasing the subsidy rate weakens the self-selection constraint but at the same time distorts the decision-making of  $x$  versus  $z$  for both types of households. For a given  $g$ , the optimal subsidy rate will balance these offsetting benefits and costs.

## 5. THE WELFARE EFFECTS OF PUBLIC PROVISION

Next, consider the effects of introducing  $g$ , given any value of the subsidy  $s$ . Equation (13) gives the welfare effect of increasing  $g$ . Note first that if neither low- nor high-ability households nor the mimicker are crowded out, then  $\partial W/\partial g = 0$ . In this case, each of the first three terms is zero by the household first-order conditions (1) and (1'), given that  $b_1, b_2, \hat{b}_2 > 0$ , and  $A_1 = A_2 = 0$ . Intuitively, public provision will be ineffective as long as no household is crowded out; it acts simply as an equal lump-sum subsidy to all households.

As  $g$  is increased, eventually some households will be crowded out. Recall from (5) and (6) that the mimicker always demands less than the two non-mimicking ability-types of households for any value of  $s$ .<sup>11</sup> Therefore, the mimicker gets crowded out at a lower values of  $g$  than the other households. Denote by  $g_1(s), g_2(s)$  and  $\hat{g}_2(s)$  the values of  $g$  at which each of the three households get crowded out, given  $s$ . Formally,  $g_i(s)$  is defined as the value of  $g$  such that  $b_i(\bar{p}, g, Y_i(s, g), B_i(s, g)) \geq 0$  becomes binding where  $Y_i(s, g)$  and  $B_i(s, g)$  are the values of before-and after-tax income when the optimal non-linear income tax is in place. Then  $\hat{g}_2(s) < \min\{g_1(s), g_2(s)\}$  for all values of  $s$ . If  $g$  is increased such that  $\hat{g}_2(s) < g \leq \min\{g_1(s), g_2(s)\}$ , expression (13) reduces to

$$\frac{\partial W(s, g)}{\partial g} = -\mu(\hat{U}_z^2 - \bar{p}\hat{U}_x^2)$$

This will be positive since by (1') the right-hand side is positive when  $g > \hat{g}_2(s)$ . Intuitively, public provision makes the mimicker worse off by constraining him to consume more than

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<sup>11</sup> We are ruling out  $s < 0$ , that is, a tax on  $z$  by the arguments of the previous section.

he would like, while leaving the welfare of the other two household types unchanged. Thus, by relaxing the self-selection constraint, a welfare improvement is made possible. This result is summarized in the following proposition:<sup>12</sup>

**Proposition 2:** *For any value of  $s$ , if  $z$  is a substitute for leisure, social welfare will be independent of  $g$  in the range  $0 \leq g \leq \hat{g}_2(s)$  and strictly increasing in  $g$  in the range  $\hat{g}_2(s) < g \leq \min\{g_1(s), g_2(s)\}$ .*

Note an important difference between Propositions 1 and 2. By Proposition 2, given our assumption that  $z$  is substitutable for leisure, public provision of  $z$  will be welfare-improving regardless of the value of  $s$ . Thus, it will *always* be useful to include public provision in the policy mix. However, by Proposition 1, a subsidy will only be welfare-improving if  $b_1 > 0$ , that is if  $g < g_1(s)$ . This will be useful in helping us to determine the optimal mix of policies.

For  $g > \min\{g_1(s), g_2(s)\}$ , some terms on the right-hand side of (13) become negative in sign. Once households of type  $i$  are crowded out, the term  $(U_z^i - \bar{p}U_x^i)$  becomes negative by (2), and its absolute size increases as  $g$  increases. As well,  $A_i$  goes from zero to unity. Eventually, this will cause  $\partial W/\partial g$  to change from positive to negative. This will reflect a local maximum in  $g$ . It may not be a global maximum since it is well-known that second-best problems are not guaranteed to satisfy standard convexity properties. For our purposes, that is not an issue and our qualitative results do not depend upon it. Suppose for illustrative purposes that we simply assume that the function  $W(s, g)$  is strictly concave in  $g$  for values of  $g \geq \hat{g}_2$ . This implies that the curve relating  $W(s, g)$  to  $g$  for given  $s$  can be depicted as in Figure 1. Social welfare is independent of  $g$  up to  $\hat{g}_2(s)$ . It then rises monotonically to a peak at point  $g^*$  somewhere beyond  $\min\{g_1(s), g_2(s)\}$ . If  $g_2(s) > g_1(s)$ , it will be the case that  $g^* > g_1(s)$ , but  $g^*$  could be above or below  $g_2(s)$ . Similarly, if  $g_1(s) > g_2(s)$ ,  $g^* > g_2(s)$ , but  $g^*$  could be above or below  $g_1(s)$ .

These results reflect the different ways in which the subsidy and public provision affect social welfare. Both policies mitigate the self-selection constraint by worsening the mimicker's utility, provided  $z$  is a substitute for leisure. But  $s$  will distort type  $i$ 's choice of  $x$  and  $z$ , while still allowing the mimicker to maximize his utility on the budget constraint. On the other hand, public provision is equivalent to lump-sum transfer of  $\bar{p}b$  if  $b_i$  is not perfectly crowded out, which implies that it does not distort individual choice of  $x$  and  $z$  insofar as  $b_i > 0$ . Since  $z$  is substitute for leisure, the mimicker's consumption is crowded out first. By providing slightly more  $g$  than  $\hat{g}_2(s)$ , the government can always discourage mimicking behaviour, without distorting the choice of  $x$  and  $z$  made by type  $i$  ( $i=1, 2$ ). Public provision can be said to be more effective than the subsidy once the mimicker gets crowded out in the sense that it can determine the consumption by the mimicker: the mimicker is not allowed to choose  $x$  and  $z$  on his own.

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<sup>12</sup> This proposition is analogous to Corollary 3 of Theorem 4 in Guesnerie (1995), p. 210, though his analysis focuses on the case of linear taxes. It is a generalization of Proposition 3 of Boadway and Marchand (1995), who assumed  $s = 0$ , but did allow for non-linear income taxation.

## 6. THE MIX OF SUBSIDY AND PUBLIC PROVISION

On the basis of the results in the previous section, we now consider when it would be optimal to use either a subsidy or public provision on its own or a mixture of the two policies. In fact, we can readily deduce from Proposition 2 the following:

**Corollary 2.1:** *The optimum would never involve using a subsidy without public provision.*

To see this, recall that if it is optimal to impose a subsidy on  $z$ ,  $z$  must be a substitute for leisure (Edwards *et al* (1994), Nava *et al* (1996)). However, from Proposition 2, if  $z$  is a substitute for leisure, it is welfare-improving to provide  $g$  at least up to the minimum level at which one of the two ability types is crowded out,  $\min\{g_1(s), g_2(s)\}$ . Therefore, whenever  $s$  is used, so would public provision be used.

That leaves open the question as to whether  $g$  would be used alone or whether a mix of  $s$  and  $g$  is optimal. To investigate that, suppose that both the optimal non-linear income tax and the optimal subsidy are in place, and consider the effect on social welfare from changes in  $g$ . For any given value of  $g$ , the optimal tax and subsidy policy will now be given by extending problem (7) to include  $s$  as a control variable. The first-order conditions now include (8) – (11) as well as the optimality condition on the choice of  $s$ , which is that equation (12) be set equal to zero. From the latter we obtain that the optimal choice of  $s$  is given by  $s^*(g)$  in (14). The maximum value function to this tax and subsidy problem of the government may now be written as  $W(s^*(g), g)$ . By the envelope theorem:

$$\frac{dW(s^*(g), g)}{dg} = \frac{\partial W(s^*(g), g)}{\partial g} \quad (15)$$

Therefore, (13) can be used to indicate how social welfare changes with  $g$ , assuming  $s = s^*(g)$ .

Using (13) and (14), we can then trace out the path of social welfare as  $g$  changes, assuming that  $s$  is always maintained at its optimal level. Let  $g_i^*$  be the value of  $g$  such that a household of type  $i$  is just crowded out, given that the subsidy is chosen optimally ( $s = s^*(g)$ ), that is, the value of  $g$  such that  $b_i(p - s^*(g), g, Y_i(s^*(g), g), B_i(s^*(g), g)) \geq 0$  becomes binding with  $\hat{g}_2^*$  being defined similarly.

For expositional purposes, we assume that both  $s^*(g)$  and  $g_i^*$  are uniquely defined and that the function  $W(s^*(g), g)$  is strictly concave for  $g \geq \hat{g}_2^*$ . Figure 2 depict four possible situations. In panels A and B, it is assumed that  $b_2 > b_1$  so that low-ability households are always crowded out before high-ability households ( $g_1^* < g_2^*$ ). In this case, it is never optimal to use a subsidy. The reason is that, by (14), the optimal subsidy is positive up to  $g_1^*$  and zero beyond that. But social welfare  $W(\cdot)$  will still be increasing in  $g$  at  $g = g_1^*$ . Therefore, the maximum value of  $W(\cdot)$  must lie in the range beyond  $g = g_1^*$  where  $s^* = 0$ . Whether both types of households are crowded out in the optimum depends on the parameters of the problem. Panels A and B depict the situations where high-ability households are not and are crowded out, respectively. Note that these qualitative results are not affected by relaxing the strict concavity assumption. If there are multiple local optima in  $g$ , all will lie in the range where  $g > g_1^*$ .

Panels C and D depict the case where  $g_1^* > g_2^*$ . Again, for  $g > g_1^*$ , the optimal subsidy will be zero. But it will be positive as long as  $g < g_1^*$ . Two outcomes are possible here. In panel C, the maximum value of  $W(\cdot)$  is obtained in the range  $g_2^* < g < g_1^*$ . In this case, at the optimum, there will be a combination of subsidy and public provision. Panel D depicts the case where  $g > g_1^*$  in the optimum. Here, both types are crowded out, so a subsidy will be ineffective. These results can be summarized as follows:

**Proposition 3:** *If  $z$  is a substitute for leisure, the optimal policy will involve  $s = 0$  if i)  $g_1^* \leq g_2^*$  or if ii)  $g_2^* \leq g_1^*$  and  $\partial W(s^*(g_1^*), g_1^*)/\partial g > 0$ . A mix of  $s > 0$  and  $g > 0$  will be optimal if  $g_2^* \leq g_1^*$  and  $\partial W(s^*(g_1^*), g_1^*)/\partial g < 0$ .*

Finally, suppose that for some reason, such as administrative costs, the government is restricted to using either a subsidy or public provision, but not both. The following Proposition applies:

**Proposition 4:** *Replacing any value of  $s$  with public provision sufficient to crowd out low-ability households will be welfare-improving if  $b_2(\bar{p}, 0, Y_2(s, 0), B_2(s, 0)) \geq b_1(\bar{p}, 0, Y_1(s, 0), B_1(s, 0))$ .*

To show this, suppose that initially  $g = 0$  and the optimal income tax is in place for a given  $s$ . We can write  $b_i^* = b_i(\bar{p}, 0, Y_i(s, 0), B_i(s, 0))$ , and define  $\hat{b}_2^*$  analogously. Now consider an increase in  $g$  by  $b_1^*$  and abolish the subsidy. Suppose that the tax payment for type  $i = 1, 2$  is changed by  $pb_1^* - sb_i^*$ , while there is no change in before-tax income,  $Y_i(s, 0)$ .  $pb_1^*$  is an increase in the cost of public provision, while  $sb_i^*$  corresponds to a reduction of subsidy payment. It is straightforward to see that the initial consumption bundles,  $(x_i, z_i)$  is still feasible after the change: the assumption of  $b_2^* \geq b_1^*$  ensures this for type 2. It is also easy to verify that government budget balance is satisfied. Now what happens to the households' welfare? Type 1 chooses not to supplement  $z$  because  $MRS_{zx}^1 = \bar{p} < p$  at  $b_1^*$ . He would prefer to decrease  $z$  below  $g = b_1^*$ , which is not possible. Therefore, this type selects exactly the same bundle of  $x$  and  $z$  as before and his utility is unchanged. For the mimicker, initially,  $b_1^*$  is feasible (he faces the same budget constraint as type 1 for given before- and after-tax incomes). After the change, the mimicker is forced to consume  $g = b_1^*$ : the mimicker always prefers to consume less  $z$  than type 1, so he does not supplement  $z$ . The revealed preference argument tells us that by this change, the mimicker is made worse off. Type 2 may still supplement  $z$  from the market. But, since the initial bundle of  $x$  and  $z$  is feasible after the change, we can safely say that at least he is not worse off. Rather, we can see that he will be strictly better off if  $b_1^* > b_2^*$ . Initially,  $MRS_{zx}^2 = \bar{p}$  at  $b_2^*$ . After the change, he is induced to decrease his consumption of  $z$  because its price is now  $p > \bar{p} = p - s$ . Again, by revealed preference, his utility increases. To summarize, this change is feasible since the government's budget constraint is satisfied and neither type is worse off, while the self-selection constraint is relaxed. Of course, Proposition 4 will apply when  $s$  is set optimally so  $s = s^*(0)$ .

In the following sections, we indicate how the analysis can be applied to two alternative cases. First, the analysis is generalized to the case where there are more than two ability-types. Next, we consider the possibility that households can choose whether or not to accept public provision that is provided out of federal general revenues, on the

understanding that if they do not, they cannot supplement public provision with private purchases.

## 7. MANY ABILITY-TYPES

Assume now that there are  $M$  ability-types indexed  $i = 1, \dots, M$  with wage rates  $w_i$  such that  $w_i < w_{i+1}$ . As above, each household type has an indirect utility function of the form  $V^i(\bar{p}, g, Y_i, B_i)$ . The single-crossing property is assumed to be satisfied so that  $MRS_{YB}^i > MRS_{YB}^{i+1}$ ,  $i = 1, \dots, M-1$ . It is well-known that the non-linear tax optimum may either be a separating equilibrium or there may be partial pooling.<sup>13</sup> That is, the same income may be chosen by adjacent ability-types. For simplicity, we assume that there is no partial pooling in our problem. This does not affect our results, all of which apply for the partial-pooling case as well (assuming of course that the single-crossing property always applies). The optimal income tax outcome is a separating equilibrium with the self-selection constraint binding with next-lowest ability type for all ability types,  $i \geq 2$ . The problem for the government, given  $s$  and  $g$  is the following generalization of problem (7):

$$\max_{\{Y_i, B_i\}} V^1(\bar{p}, g, Y_1, B_1) + \sum_{i=2}^M \lambda_i V^i(\bar{p}, g, Y_i, B_i) \quad (7')$$

subject to

$$V^i(\bar{p}, g, Y_i, B_i) \geq \hat{V}^i(\bar{p}, g, Y_{i-1}, B_{i-1}) \quad i = 2, \dots, M \quad (\mu_i)$$

$$\sum_{i=1}^M N_i(Y_i - B_i - sb_i - pg) \geq 0 \quad (\gamma)$$

where the  $\mu_i$ 's now refer to the Lagrange multipliers on the  $M-1$  self-selection constraints. The first-order conditions are obvious generalizations of (8) – (11), and the interpretation of them in terms of marginal tax rates are similar. For our purposes, the more interesting results concern the effects of  $s$  and  $g$ . Defining the maximum value function for problem (7') by  $W(s, g)$  as before, application of the envelope theorem gives the following analogues to (12) and (13):

$$\frac{\partial W(s, g)}{\partial s} = \sum_{i=2}^M \mu_i \hat{U}_x^i(b_{i-1} - \hat{b}_i) + \lambda s \sum_{i=1}^M N_i \frac{\partial \tilde{b}_i}{\partial \bar{p}} \quad (12')$$

and

$$\frac{\partial W(s, g)}{\partial g} = (U_z^1 - \bar{p}U_x^1) + \sum_{i=2}^M (\lambda + \mu_i)(U_z^i - \bar{p}U_x^i) - \sum_{i=2}^M \mu_i(\hat{U}_z^2 - \bar{p}\hat{U}_x^2) - \lambda s \sum_{i=1}^M N_i A_i \quad (13')$$

where  $A_i = 0$  if  $b_i > 0$  and  $A_i = 1$  if  $b_i = 0$  as before. From (12') and (13'), we can obtain generalizations of Propositions 1–3.

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<sup>13</sup> See Guesnerie and Seade (1982).



Given that  $z$  is a substitute for leisure, we know that  $b_{i-1} \geq \hat{b}_i$  for all  $i \geq 2$  as in (5). Therefore, given the amount of public provision  $g$ , from (12') at  $s = 0, \partial W(s, g)/\partial s > 0$  if there is at least one type  $i \leq M - 1$  that is not crowded out by public provision ( $b_i > 0$ ). At the same time, if all  $b_i = 0, i = 1, \dots, M - 1$ , the optimal subsidy will be zero. This generalizes Proposition 1 to the many-person case.

Next, from (13'), we see that for any  $s$ ,  $W(s, g)$  is independent of  $g$  as long as no mimicker is crowded out. As soon as any one mimicker is crowded out, social welfare will be strictly increasing in  $g$  at least until one household becomes crowded out. Thus, as in the two-person case, it is always optimal to employ some public provision. This generalizes Proposition 2. Moreover, since this result holds for any level of  $s$ , Corollary 2.1 also applies to the many-person case: the optimum would never involve using a subsidy without public provision.

As for Proposition 3, suppose we evaluate (13') for various values of  $g$  assuming that  $s$  is chosen optimally. As before, let  $s^*(g)$  be the value of  $s$  that makes (12') equal zero and assume that  $s^*(g)$  is uniquely valued. Then, (15) yields the effect of  $g$  on social welfare. We can trace out the path of social welfare as  $g$  changes, with  $s = s^*(g)$ . If  $W(s^*(g), g)$  is strictly concave above the point at which the first mimicker is crowded out, its shape will be similar to that of Figure 2. The curve will begin rising at the level of  $g$  at which the first mimicker is crowded out and will continue to rise beyond the point at which the first non-mimicking ability type is crowded out. It will reach its peak after some indeterminate number of ability-types are crowded out. Let  $g_i^*$  be the value of  $g$  at which households of ability-type  $i$  become crowded out, given that the subsidy is chosen optimally,  $s = s^*(g)$ . In the two-type case, we found that if  $g_2^* > g_1^*$ , there would be no subsidy in the optimum. This may not be true, however, in the present case. Suppose we assume that  $g_i^*$  is increasing in ability when there are  $M$  ability-types. Then, as shown in Figure 3, a possible outcome is that social welfare  $W(s^*(g), g)$  reaches its peak at  $g < g_{M-1}^*$ . If so, some high-ability households will not be crowded out, and for them  $b_{i-1} > \hat{b}_i$ . The value of  $s^*(g)$  that makes (12') equal to zero will be positive. Thus, when there are more than two ability-types, it is no longer the case that if lower ability-types are crowded out first ( $g_i^* > g_{i-1}^*$ ), a subsidy will not be used in the optimum. On the contrary, in the many-person case, the presumption is that both a subsidy and public provision will be used in the optimum. In other words, Proposition 3 does not generalize to the many-person case.

We can summarize the results of this section as follows:

**Proposition 5:** *In an economy with  $M$  ability-types, assuming that  $z$  is a substitute for leisure and  $g_{i-1}^* < g_i^*$  for all  $i$ , if  $g^* < g_{M-1}^*$ , the optimal policy will involve both a subsidy and public provision.*

Note the following differences between  $s$  and  $g$ , analogous to those found for the two-type case. First, public provision is effective for relaxing the self-selection constraints of those mimickers whose consumption of  $z$  is perfectly crowded out: the mimicker is forced to consume a particular bundle of  $x$  and  $z = g$  that they would not have otherwise chosen. Public provision cannot be effective for the self-selection constraints involving the mimickers whose consumption of  $z$  is not yet crowded out. But, the self-selection

constraints involving these mimickers can be affected by the subsidy. Of course it may not be desirable to crowd out all types' consumption of  $z$  since the decrease in welfare of lower ability types due to further increase in  $g$  (recall  $U_z^i - \bar{p}U_x^i < 0$ ) may be significant, which leads to our conclusion.

## 8. PUBLIC AND PRIVATE PROVISION MUTUALLY-EXCLUSIVE

Blomquist and Christiansen (1995) have considered the case in which households may choose to accept public provision, but if they do 'opt in' they are unable to supplement public with private provision. In this case, if a household of type  $i$  opts in, so  $b_i = 0, z_i = g$ : if the household does not opt in,  $z_i = b_i$ . Following Blomquist and Christiansen, we initially assume that there is no charge for  $g$  if a household opts in. Later we allow for a user charge.<sup>14</sup> Moreover, income taxation cannot be made contingent on whether a household opts in. That is, the government cannot identify the income of persons who opt for public provision. As in the basic model, this informational constraint is reasonable if households decide whether to opt in or out before income is reported.

If a household of type  $i$  opts out of consuming the public good, indirect utility is given by  $V^i(\bar{p}, 0, Y_i, B_i)$  defined in (2) above. Underlying this indirect utility function is the demand function for  $z$ ,  $b_i(\bar{p}, 0, Y_i, B_i)$ , which satisfies the properties set out earlier. On the other hand, if the household opts in, utility is given by  $U^i(B_i, g, Y_i)$ . The household will choose to opt in if  $U^i(B_i, g, Y_i) > V^i(\bar{p}, 0, Y_i, B_i)$ . The analogous utility functions for the mimicking household who opt out and in are  $\hat{V}^2(\bar{p}, 0, Y_1, B_1)$  and  $\hat{U}^2(B_1, g, Y_1)$ . It can be shown that whenever a household of type 1 chooses to opt in, so will the mimicker. That is, for any  $\{Y_1, B_1\}$ ,  $s$ , and  $g$ , if  $U^1(B_1, g, Y_1) \geq V^1(\bar{p}, 0, Y_1, B_1)$ , then  $\hat{U}^2(B_1, g, Y_1) > \hat{V}^2(\bar{p}, 0, Y_1, B_1)$ .

We wish to consider whether a subsidy would be used in the optimum when it is no longer possible to supplement public provision with private purchases of  $z$ . There are three possible cases. In the first two (cases (1) and (2) below), public provision is set at a level such that low-ability households choose to opt in. Then, mimickers will also opt in. In case (1), the high-ability households opt out and in case (2) they opt in. In case (3), no public provision is undertaken so obviously all households have no option but to opt out. Since switching from one regime to another by increasing  $g$  involves a discrete change, differential analysis cannot be used. Instead, we analyze each of the three cases separately to discover the optimal policy combination and the level of social welfare attained. In considering each of the cases, we must ensure that those who are taken as opting out prefer to opt out and

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<sup>14</sup> Munro (1992) has considered the case in which opting in is accompanied by a user charge. However, income taxation is restricted to being linear. Since this is inefficient, it provides an extraneous reason for in-kind transfers to be welfare-improving. The case in which public and private provision are mutually exclusive is also considered by Besley and Coate (1991). They let the quality of the publicly provided good to differ from that chosen by those who opt out. However, their analysis assumes proportional income taxation. Few of the models used in the literature consider subsidization as an alternative policy instrument. An exception is Nichols and Zeckhauser (1982).

similarly for those who are taken as opting in. This will add additional constraints that must be satisfied by the government in choosing its policy instruments for each case.

Case (1): Type 1's opt in; type 2's opt out

Consider first the optimal income tax and subsidy policy, given the value of  $g$ . The problem of the government may be written as follows:

$$\max_{\{Y_i, B_i, s\}} U^1(B_1, g, Y_1) + \lambda V^2(\bar{p}, 0, Y_2, B_2)$$

subject to

$$V^2(\bar{p}, 0, Y_2, B_2) \geq \hat{U}^2(B_1, g, Y_1) \quad (\mu)$$

$$N_1(Y_1 - B_1 - pg) + N_2(Y_2 - B_2 - sb_2) \geq 0 \quad (\gamma)$$

$$U^1(B_1, g, Y_1) \geq V^1(\bar{p}, 0, Y_1, B_1) \quad (\phi_1)$$

$$V^2(\bar{p}, 0, Y_2, B_2) \geq U^2(B_2, g, Y_2) \quad (\phi_2)$$

where again the equation labels refer to the Lagrange multipliers. The last two constraints restrict policy options to those that ensure that low-ability households opt in and high-ability households opt out, respectively. We refer to these as *participation constraints*. The solution to this problem gives us the optimal non-linear tax and subsidy policies. In what follows,  $(Y_i, B_i), i = 1, 2$  are assumed to satisfy the corresponding first-order conditions, and we concentrate on the choice of  $s$  and  $g$ .

The optimal subsidy follows from the first-order conditions on  $B_2$  and  $s$ :

$$(\lambda + \mu)U_x^2 - \gamma N_2 \left( 1 + s \frac{\partial b_2}{\partial B_2} \right) + \phi_2 U_x^2 - \bar{U}_x^2 = 0 \quad (9')$$

$$(\lambda + \mu + \phi_2)U_x^2 b_2 - \gamma N_2 \left( b_2 - s \frac{\partial b_2}{\partial \bar{p}} \right) - \phi_1 \bar{U}_x^1 \bar{b}_1 = 0 \quad (12')$$

where  $\bar{U}_x^1$  refers to the marginal utility of  $x$  if household 1 opts out,  $\bar{U}_x^2$  the marginal utility of  $x$  if household 2 opts in, and  $\bar{b}_1$  is the demand for  $z$  by households of type 1 if they opt out. Combining (9') and (12') and solving for  $s$  yields:

$$s = - \frac{\phi_2 b_2 \bar{U}_x^2 - \phi_1 \bar{b}_1 \bar{U}_x^1}{\gamma N_2 \partial \bar{b}_2 / \partial \bar{p}} \quad (16)$$

Since the denominator is negative, (16) implies that  $s = 0$  if  $\phi_1 = \phi_2 = 0$ , so neither participation constraint is binding. If  $\phi_1 = 0$  and  $\phi_2 > 0$ , then imposing a subsidy is optimal ( $s > 0$ ). But if the participation constraint on the type-1 households alone is binding so  $\phi_1 > 0, \phi_2 = 0$ , then  $s < 0$ , that is, it is optimal to impose a *tax* on private purchases of  $z$ . The intuition for these results is straightforward. The subsidy or tax  $s$  is useful for ensuring that the participation constraints are satisfied. If they are not binding,  $s$  should not be used since it simply distorts the choice of  $z$  by the high-ability households.

If  $\phi_1 = 0$  and  $\phi_2 > 0$ , however, a positive subsidy will induce them to opt out of public provision. If the constraint on the low-ability households is binding, there must be taxes on their demand for  $z$  in order to ensure that they opt in. Of course, if both constraints are binding, the sign of  $s$  is ambiguous: the desire to satisfy both constraints leads to offsetting influences on the subsidy rate.

Denote the maximum value function associated with this case (1) problem by  $W^1(g)$ . It is the welfare that is achieved at various values of  $g$  given that  $s$  is chosen optimally and that only high-ability households opt out of public provision. We can determine the effect of  $g$  on social welfare by differentiating the Lagrange expression for this problem with respect to  $g$  and using the first-order condition on  $B_1$ :

$$\frac{\partial W^1(g)}{\partial g} = (1 + \phi_1)(U_z^1 - \bar{p}U_x^1) - \mu(\hat{U}_z^2 - \bar{p}\hat{U}_x^2) + \bar{p}\phi_1\bar{U}_x^1 - \phi_2\bar{U}_z^2 - \gamma N_1 s \quad (17).$$

In the optimum for case (1), (17) will be zero. Denote the policies that give the global maximum in this case as  $(g^{1*}, s^{1*})$  and the maximized value of social welfare as  $W^{1*}$ . Depending on the parameters of the economy,  $s^{1*}$  could be positive, negative or zero depending upon which participation constraints, if any, are binding.

Case (2): Both types opt in

The government problem now becomes:

$$\max_{\{Y_i, B_i, s\}} U^1(B_1, g, Y_1) + \lambda U^2(B_2, g, Y_2)$$

subject to

$$U^2(B_2, g, Y_2) \geq \hat{U}^2(B_1, g, Y_1) \quad (\mu)$$

$$N_1(Y_1 - B_1 - pg) + N_2(Y_2 - B_2 - pg) \geq 0 \quad (\gamma)$$

$$U^1(B_1, g, Y_1) \geq V^1(\bar{p}, 0, Y_1, B_1) \quad (\phi_1)$$

$$U^2(B_2, g, Y_2) \geq V^2(\bar{p}, 0, Y_2, B_2) \quad (\phi_2).$$

The last constraint now reflects the requirement that the high-ability persons prefer to opt in. Since the subsidy  $s$  appears only in the two participation constraints, and since reductions in  $s$  tend to relax both constraints, setting the value of  $s$  sufficiently low (possibly negative) will insure that both participation constraints are satisfied and that no private purchases of  $z$  are ever made by either household type.<sup>15</sup> Any subsidy or tax would only be notional since it serves only as a threat to enforce all households to opt in.

Denote the social welfare achieved in this case by  $W^2(g)$ . The effect of changes in  $g$  on social welfare is given by applying the envelope theorem to the case (2) problem. Using the first-order conditions and  $\phi_i = 0$  this yields:

$$\frac{\partial W^2(g)}{\partial g} = (U_z^1 - pU_x^1) + (\lambda + \mu)(U_z^2 - pU_x^2) - \mu(\hat{U}_z^2 - p\hat{U}_x^2) \quad (18)$$

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<sup>15</sup> Alternatively, the government could presumably make it illegal to supply  $z$  for private sale.

In the global maximum for case (2), (18) will be set to zero. The maximizing solutions for the policy variables will be denoted  $(g^{2*}, s^{2*})$ , and maximum social welfare is  $W^{2*}$ . Let  $s_i^o(g)$  be the value of  $s$  that can be obtained by solving  $U^i(B_i^*, g, Y_i^*) = V^i(p - s, 0, Y_i^*, B_i^*)$  for  $s$ , where  $(Y_i^*, B_i^*)$  is an optimized income tax for a given  $g$ .<sup>16</sup> The optimal subsidy  $s^{2*}$  is not uniquely defined. Any value of  $s^{2*} \leq \min\{s_1^o(g^{2*}), s_2^o(g^{2*})\}$  will do.

Case (3): No public provision

The case of no public provision is the well-known one in which the government chooses a non-linear tax along with a differential commodity tax structure. Given that  $z$  is a substitute for leisure, the optimal commodity tax structure will be one in which the relative tax on  $z$  is lower than that on  $x$ , that is,  $s > 0$  (Edwards *et al* (1994), Nava *et al* (1996)). Specifically, the optimal subsidy,  $s^{3*}$ , will be given by (14) with  $g = 0$ . The maximum value attained for social welfare is denoted  $W^{3*}$ . Because the optimal choice of  $g$  in cases (1) and (2) must satisfy additional constraints that were not relevant when supplementation was possible, it is now possible that, unlike in the earlier case, the global optimum may involve a subsidy but no public provision ( $W^{3*} > \max\{W^{1*}, W^{2*}\}$ ).

In general, we cannot rule out the possibility that any of these three cases is globally optimal. We can, however, narrow the range of possibilities in two ways. First, it is straightforward to show that  $W^{3*} > W^{1*}$  if  $s^{1*} < 0$ , that is, case (1) cannot be globally optimal if it involves a tax on purchases of  $z$ .<sup>17</sup> Second, case (1) will be the global optimum if the solution to its optimization is unique and if neither participation constraint is binding ( $\phi_1 = \phi_2 = 0$ ).<sup>18</sup> We can summarize the results for this section as follows:

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<sup>16</sup> It is straightforward to show that  $s_1^o(g) > s_2^o(g)$  if  $\ell_2 > \ell_1$ ,  $B_2 > B_1$  and  $U_{xz} \geq 0$ .

<sup>17</sup> To show this, start at the case (1) optimum. Eliminate  $g$  and keep the values of  $\{Y_i^{1*}, B_i^{1*}\}$  and  $s^{1*}$  unchanged. High-ability households are equally well off. As well, the utility of low-ability households is unaffected since they were indifferent between opting out and opting in (given  $s^{1*} < 0$  so that their participation constraint is binding). Mimickers are worse off because they strictly prefer opting in when low-ability households are just indifferent. And, government net revenues will have increased by  $N_1(pg^{1*} - s^{1*}b_1)$ , the saving in public provision plus the revenues from the tax on private purchases on  $z$  (since  $s^{1*} < 0$  by assumption).

<sup>18</sup> To prove this, suppose there is no participation constraint for either type: imagine, for example, the case where the government can force either type to opt in or out depending on income observed. Then the case in which type-1 households (and the mimickers) opt in and type-2 households opt out is optimal, and  $s=0$  since it only distorts the choice of  $x$  and  $z$  by type 2. By providing  $g$  at least up to the level of  $z$  which is chosen by type 1 in the absence of public provision, the mimickers can be made worse off, which relaxes the self-selection constraint. The tax payment can be adjusted so that there is no change in welfare of either type, while satisfying the government's budget constraint. For this case, we can obtain a set of first-order conditions, which should be identical to those of case (1) with  $\phi_1 = \phi_2 = 0$ . If this set of necessary conditions for the optimization without participation constraints provides a unique solution and if  $\phi_1 = \phi_2 = 0$  in the optimum of case (1), the solution to these two optimization problems should be coincident with each other.

**Proposition 6:** *If  $z$  is a substitute for leisure, and if it is not possible to supplement public with private provision, optimal policy may be one of the following:*

- (1) *Low-ability persons opt in, high-ability persons opt out, with  $g^{1*} > 0, s^{1*} \geq 0$ ;*
- (2) *Both household types opt in, with  $g^{2*} > 0, s^{2*} \leq \min\{s_1^o(g^{2*}), s_2^o(g^{2*})\}$ ;*
- (3) *No public provision ( $g^{3*} = 0$ ), with  $s^{3*} > 0$ .*

*Case (1) will be optimal if the solution is unique and if neither participation constraint is binding.*

Unlike when public provision can be supplemented by private purchases, it is quite possible for a mix of public provision and subsidies to be optimal when the low ability type does not purchase  $z$  from the market. The reason is that subsidization might be required in order to enforce participation constraints.

The use of subsidies (or taxes) imposes a benefit or cost on those who choose to opt out. It is also useful to consider the alternative of imposing a cost or benefit on those who opt in by charging a user fee on user of  $g$ .<sup>19</sup> The relevant case is that of case (1) in which type 1 households opt in and type 2 household opt out.<sup>20</sup> Case (1) has two participation constraints. A user charge purely changes the opportunity cost of opting in: it does not affect the distribution of disposable incomes. That is, when a user charge is imposed, it is paid by low-ability types since they are the ones who opt in. But  $B_1$  can be adjusted so that their disposable income is held constant. Thus, the user charge does not affect government net revenues, but only the two participation constraints. On the other hand, the subsidy does change the net revenue of the government and, therefore, influences the optimal income tax and public provision. If neither participation constraint is binding, neither a subsidy or a user charge need be used. If one participation constraint is binding, it will be efficient to impose a user charge, positive or negative depending on the constraint. Either this will fully relax the constraint, or it will cause the other constraint to be binding first. In the former case, only a user charge is needed. In the latter, both constraints are binding in the optimum, and it is necessary to combine a positive subsidy with a user charge, which can be positive or negative.<sup>21</sup> So far we have supposed that the

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<sup>19</sup> As mentioned, Munro (1992) has considered user fees for the case in which the government is able to use linear tax schemes.

<sup>20</sup> In case (2), a user charge is essentially equivalent to a subsidy. Both can be used to induce participation by both types. In case (3) a user charge is irrelevant since there is no public provision.

<sup>21</sup> To show this, let  $G$  be the user charge. Then, disposable income by a type- $i$  household is  $B_i - G$ . The problem of the government is the same as in case (1) above except that the utility of households who opt in is given by  $U^i(B_i - G, g, Y_i)$  (and equivalently for the mimicker), and the government budget constraint is  $N_1(Y_1 - B_1 + G - pg) + N_2(Y_2 - B_2 - sb_2) \geq 0$ . The first-order condition on  $s$  reduces to (16), using the optimality conditions for the non-linear income tax. Similarly, the first-order condition on  $G$  becomes  $\phi_1 \bar{U}_x^1 = \phi_2 \bar{U}_x^2$ . Using this condition, (16) can be rewritten:  $s = -\{\phi_1 \bar{U}_x^1 (b_2 - \bar{b}_1)\} / \{\gamma N_2 \partial \bar{b}_2 / \partial \bar{p}\}$ . The results follow

mimicker always prefers to opt in, which is definitely true in the absence of a user charge. However, when the user charge is imposed and is strictly positive, it may be the case that the mimicker's participation constraint becomes binding: from a mimicker's viewpoint, by opting out, he can keep more disposal income while consuming the good  $z$  at his preferred level. The possibility of such a case may restrict the use of the user charge.<sup>22</sup>

## 9. CONCLUSION

This paper is part of a growing literature on designing efficient policies for redistribution. Ever since the seminal work by Mirrlees (1971), the role of incentive constraints as sources of limits to redistribution has been apparent. They essentially preclude the economy from attaining points along the economy's first-best utility possibilities frontier. The work of Mirrlees, followed by Stiglitz (1982) and others, characterized the utility possibilities that existed if the government used a non-linear income tax that fully exploited the incentive constraints facing the economy.

Our purpose has been to investigate the use of policy instruments alongside the optimal non-linear income tax that can improve the efficiency of redistribution by expanding the second-best utilities possibility frontier. The policies that we considered were indirect commodity taxes and quantity controls. In either case, the policies essentially worked by relaxing the economy's incentive constraints. Previous literature had dealt separately with indirect taxation and quantity controls (in-kind transfers and rationing). We assumed that the government had access to both types of instrument. In the base case we considered in which both in-kind transfers and subsidization were welfare-improving when used separately, we found perhaps unexpectedly that quantity constraints were always part of the optimal policy mix, but subsidies were not. In other cases, the policy mix varied according to the situation.

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from these conditions.

<sup>22</sup> Let  $\hat{\phi}_2$  be the multiplier of the mimicker's participation constraint. If  $\hat{\phi}_2 > 0$ , the first-order conditions in the previous footnote become:  $\phi_1 \bar{U}_x^1 + \hat{\phi}_2 \bar{\bar{U}}_x^2 = \phi_2 \bar{U}_x^2$  and  $s = - \left( \phi_1 \bar{U}_x^1 (b_2 - \bar{b}_1) + \hat{\phi}_2 \bar{\bar{U}}_x^2 (b_2 - \bar{\hat{b}}_2) \right) / (\gamma N_2 \partial \bar{b}_2 / \partial \bar{p})$  respectively, where  $\bar{\bar{U}}_x^2$  refers to the mimicker's marginal utility of income,  $\bar{\hat{b}}_2$  the demand for  $z$  if he opts out.

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